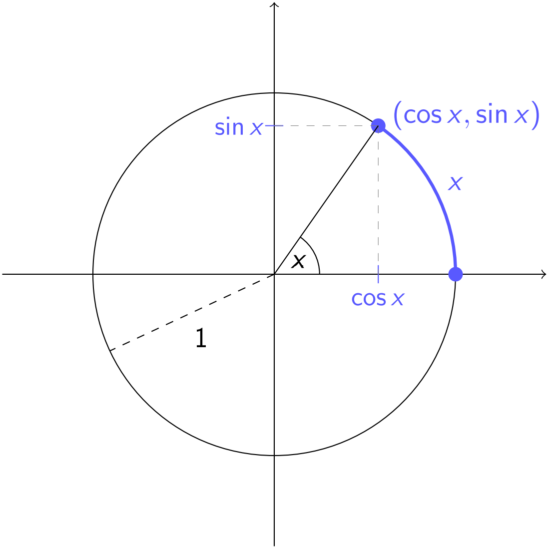
2.8. Trigonometrijske i arkus funkcije

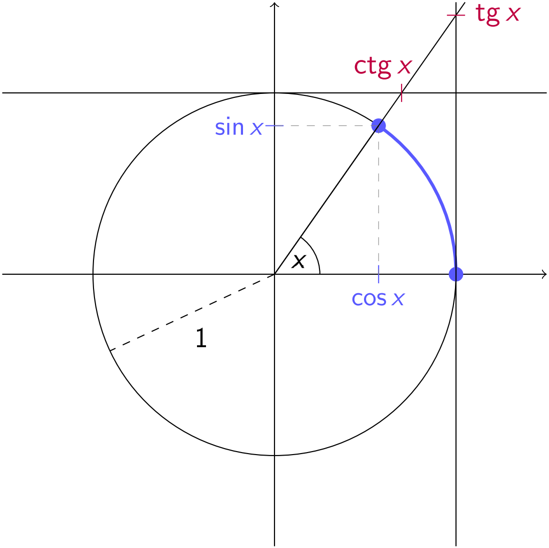
23.10. 2020.

Fotografija: Ricardo Gomez Angel

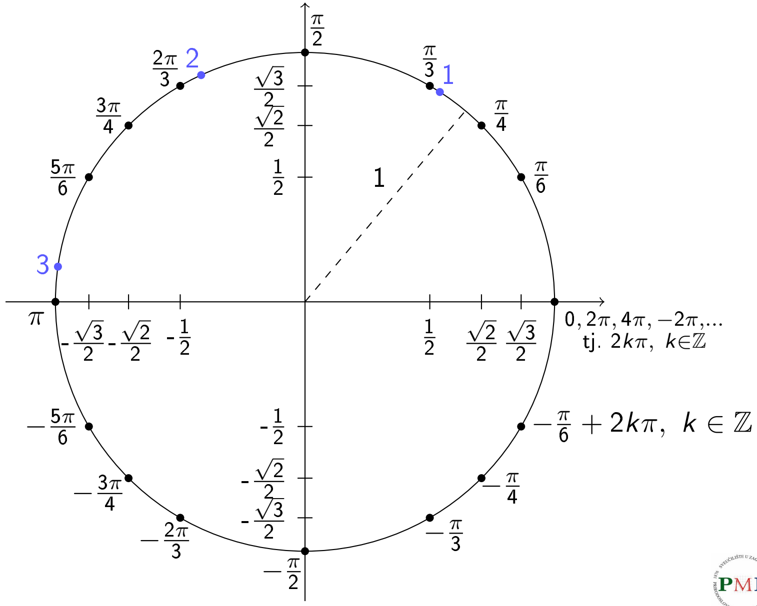
$\sin x, \cos x$



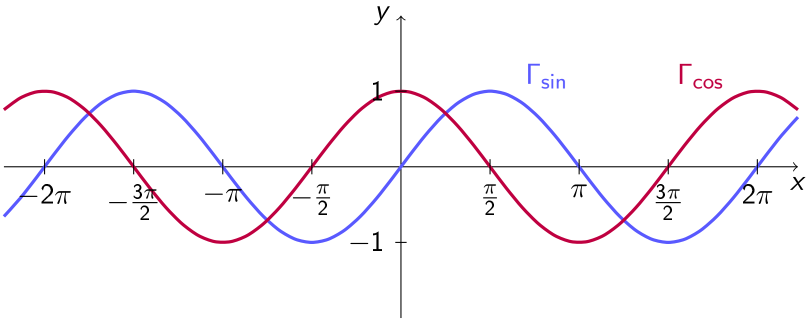
$\sin x,\ \cos x,\ \operatorname{tg} x := \frac{\sin x}{\cos x},\ \operatorname{ctg} x := \frac{\cos x}{\sin x}$



Brojevna kružnica



Funkcije sin i cos



- Funkcija $\sin : \mathbb{R} \to \mathbb{R}$ je neparna i 2π-periodična, dakle

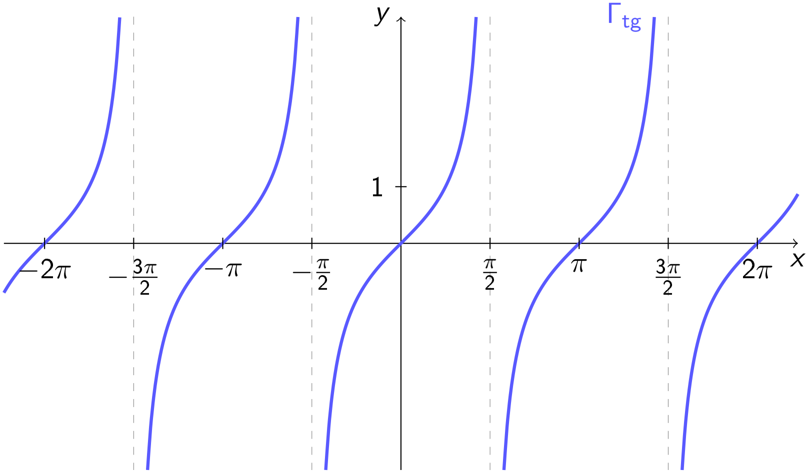
$$\sin(-x) = -\sin x \qquad \text{i} \qquad \sin(x + 2\pi) = \sin x$$

  za sve $x \in \mathbb{R}$.
- Funkcija $\cos : \mathbb{R} \to \mathbb{R}$ je parna i 2π-periodična, dakle

$$\cos(-x) = \cos x \qquad \text{i} \qquad \cos(x + 2\pi) = \cos x$$

  za sve $x \in \mathbb{R}$.

Funkcija tg

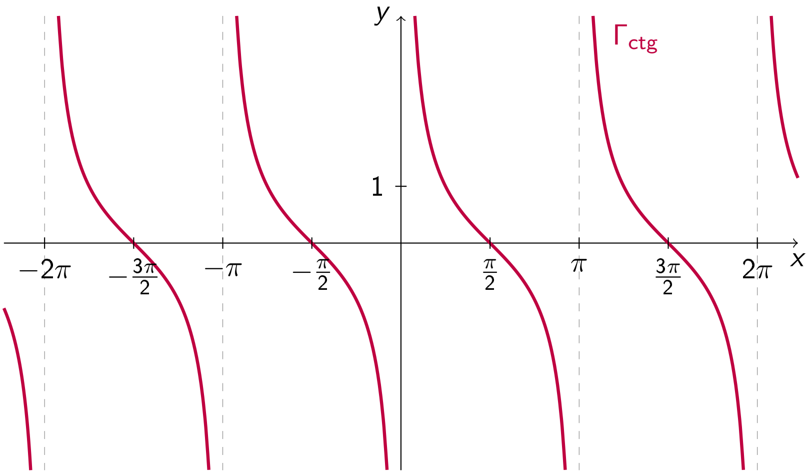


- Funkcija $\mathrm{tg} : \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\right\} \to \mathbb{R}$ je neparna i π-periodična:

$$\mathrm{tg}(-x) = -\mathrm{tg}\, x \qquad \text{i} \qquad \mathrm{tg}(x+\pi) = \mathrm{tg}\, x$$

za sve $x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\right\}$.

Funkcija ctg



- Funkcija $\text{ctg} : \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$ je neparna i π-periodična:

$$\text{ctg}(-x) = -\text{ctg}\, x \qquad \text{i} \qquad \text{ctg}(x + \pi) = \text{ctg}\, x$$

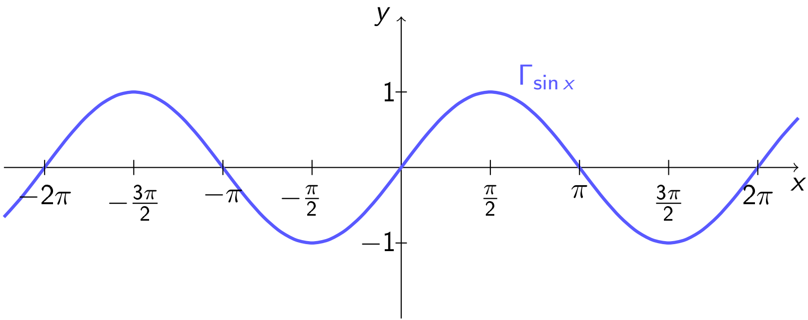
za sve $x \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.

Još neka svojstva sinusa i kosinusa

Za sve $x, y \in \mathbb{R}$ vrijedi:

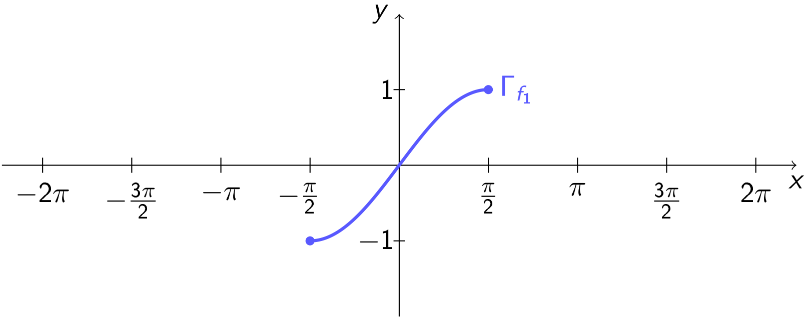
- $\sin^2 x + \cos^2 x = 1$
- $\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y$
- $\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$
- $\sin 2x = 2 \sin x \cdot \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\sin\left(x + \frac{\pi}{2}\right) = \cos x$
- $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$
- $\cos x - \cos y = -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2}$
- $\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cdot \cos \frac{x \mp y}{2}$.

Funkcija arcsin



Funkcija $f(x) := \sin x$ nije bijekcija.

Funkcija arcsin

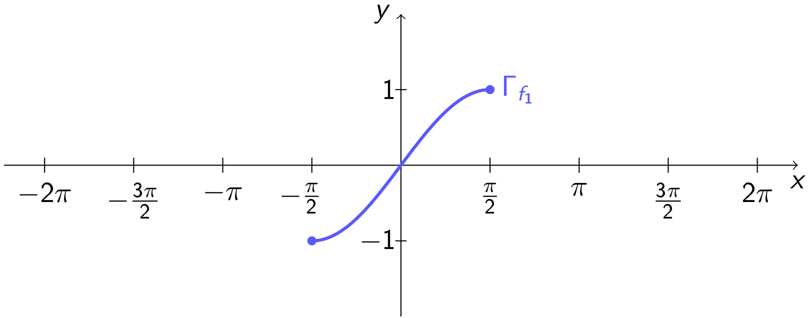


Funkcija $f(x) := \sin x$ nije bijekcija. Ali funkcija $f_1 : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$,

$$f_1(x) := \sin x,$$

jest bijekcija.

Funkcija arcsin



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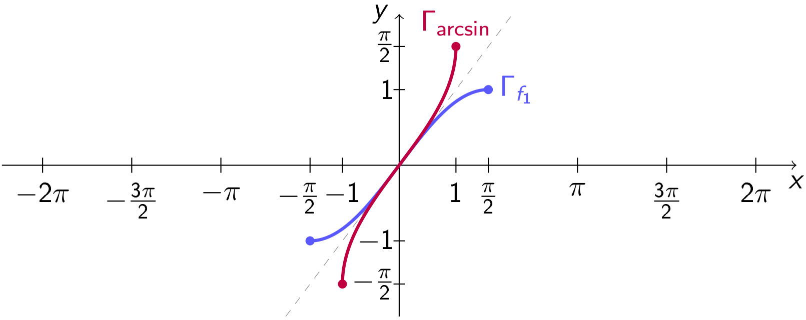
jest bijekcija. Njen inverz

$$\arcsin := f_1^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

zovemo **arkus sinusom**.

Funkcija arcsin



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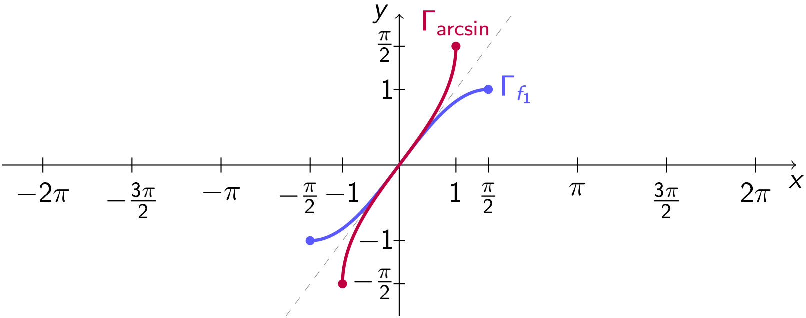

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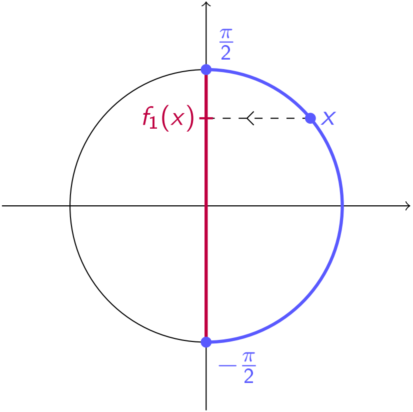
$$\arcsin := f_1^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

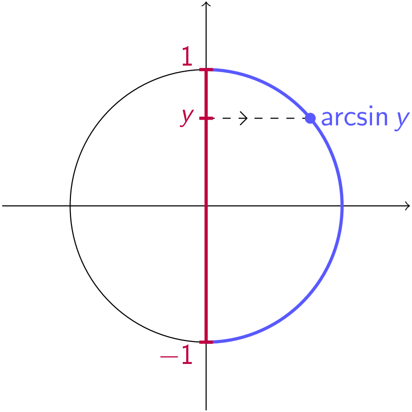
zovemo **arkus sinusom**. Zapamtimo:

$$\arcsin y = \text{jedinstven } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ takav da je } \sin x = y.$$

Funkcija arcsin na brojevnoj kružnici

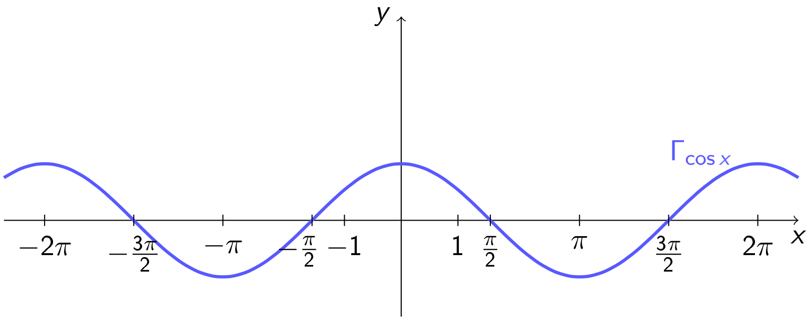


$f_1 : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$



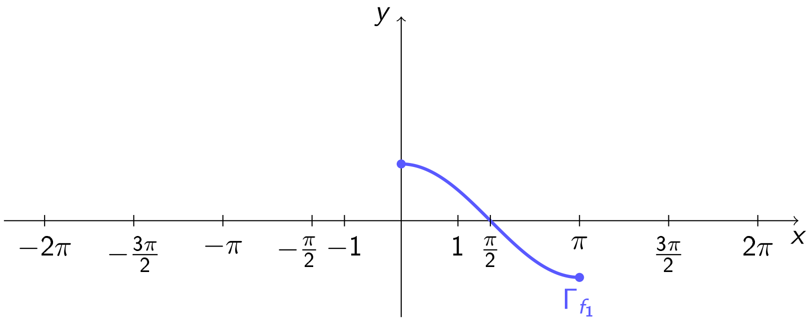
$\arcsin : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Funkcija arccos



Funkcija $f(x) := \cos x$ nije bijekcija.

Funkcija arccos



Funkcija $f(x) := \cos x$ nije bijekcija. Ali funkcija $f_1 : [0, \pi] \to [-1, 1]$,

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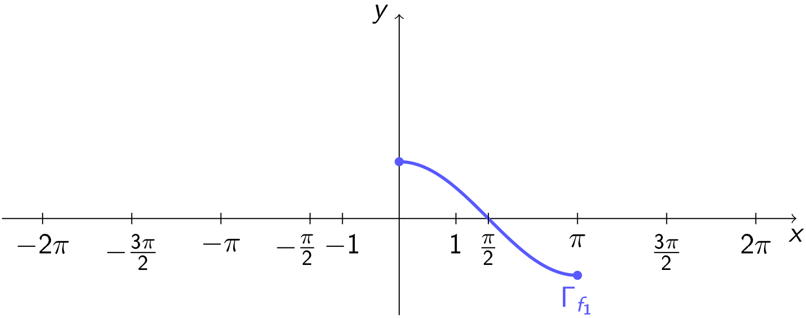
Funkcija arccos



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zovemo **arkus kosinusom**.

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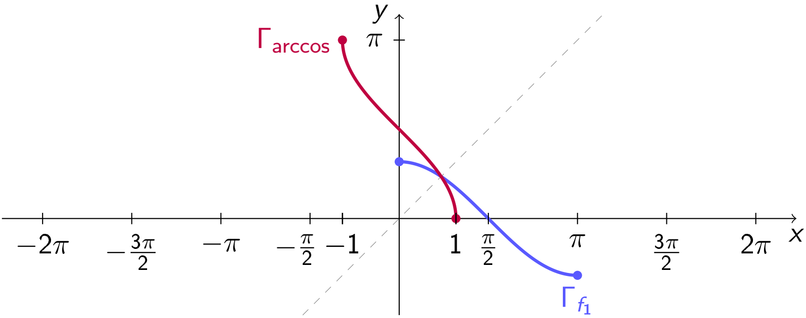

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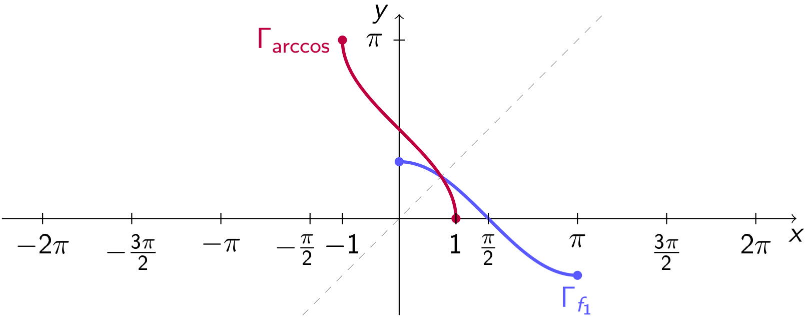

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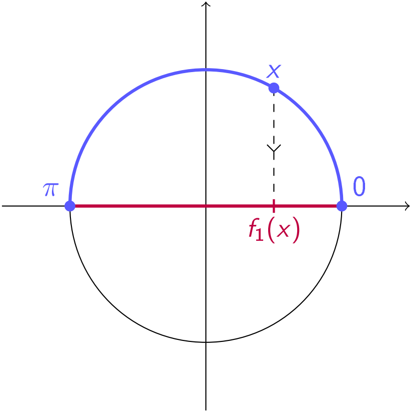
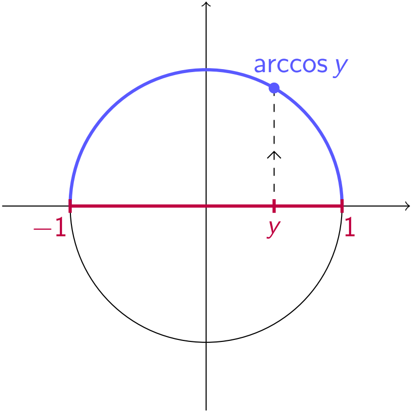
zovemo **arkus kosinusom**. Zapamtimo:

$$\arccos y = \text{jedinstven } x \in [0, \pi] \text{ takav da je } \cos x = y.$$

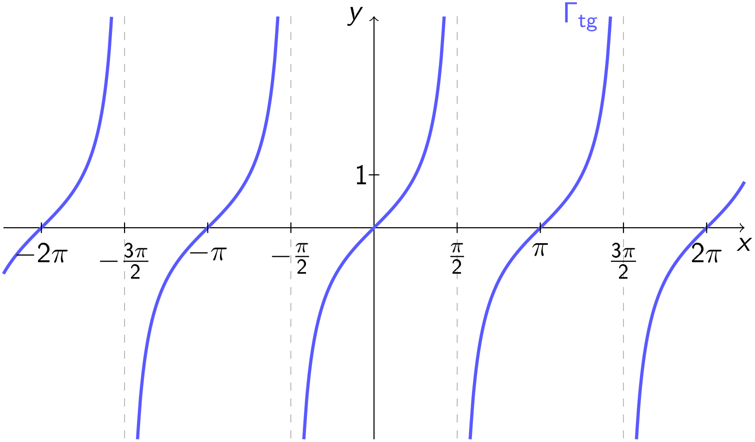


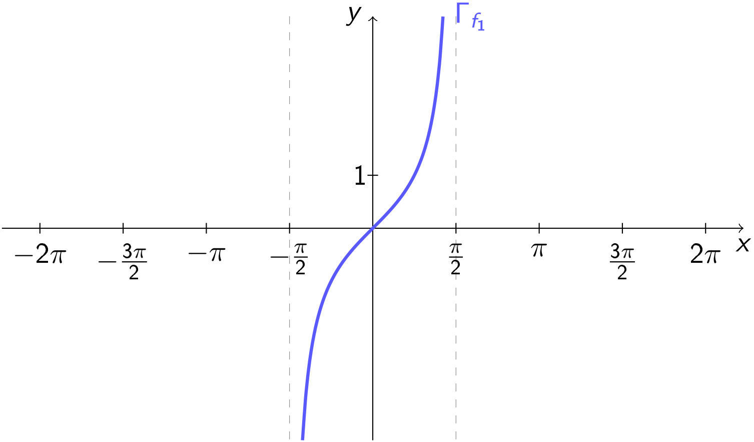


$f_1 : [0, \pi] \to [-1, 1]$

$\text{arccos} : [-1, 1] \to [0, \pi]$

Funkcija arctg





Funkcija arctg definira se kao inverz bijekcije $f_1 : \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \to \mathbb{R}$,

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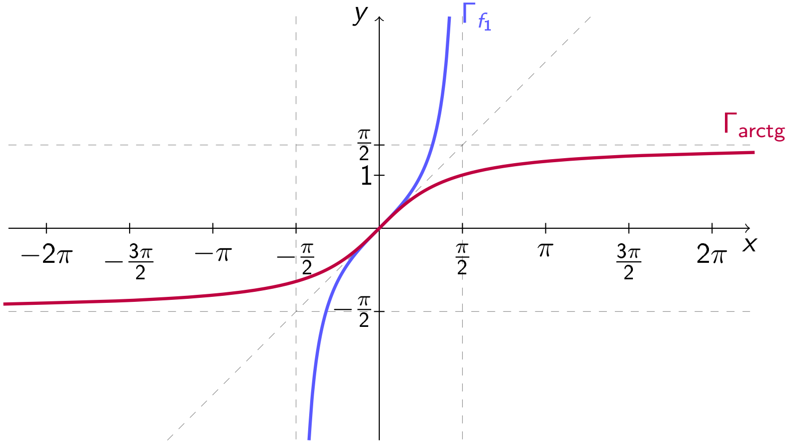
Funkcija arctg



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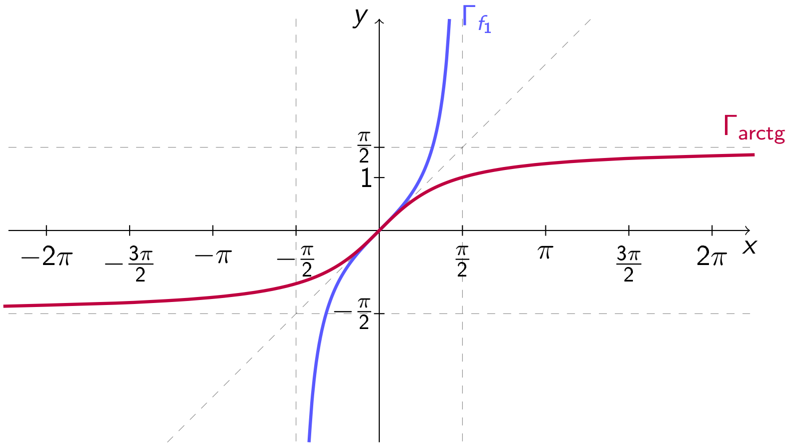

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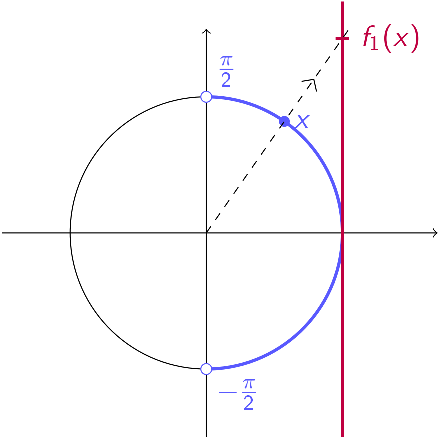
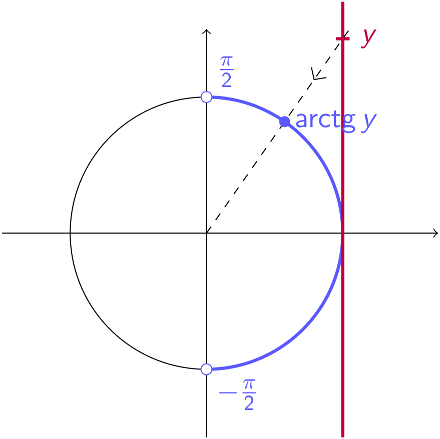
Funkcija arctg na brojevnoj kružnici

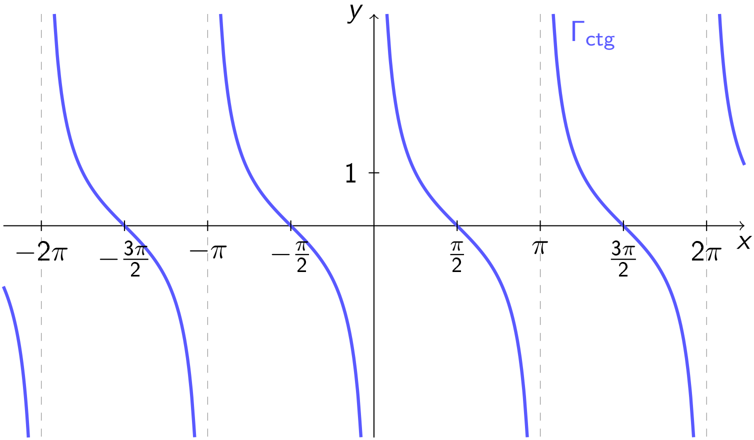




$f_1 : \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \to \mathbb{R}$

$\text{arctg} : \mathbb{R} \to \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$

Funkcija arcctg



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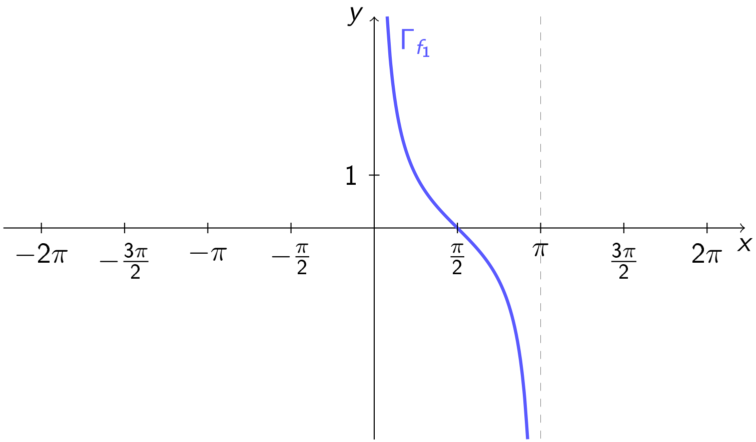

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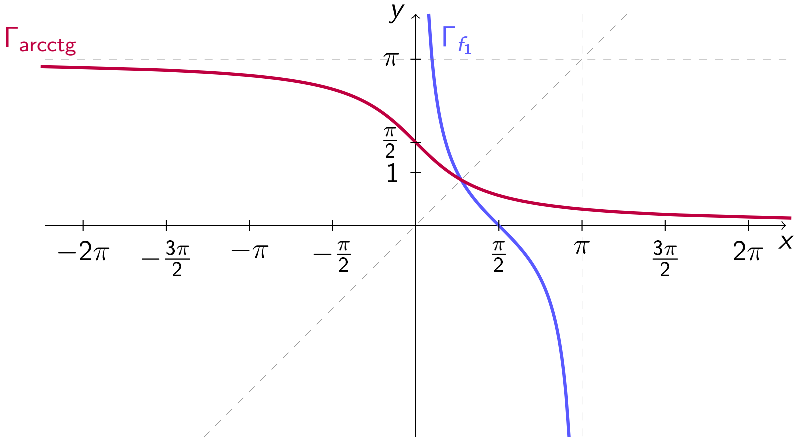
Funkcija arcctg



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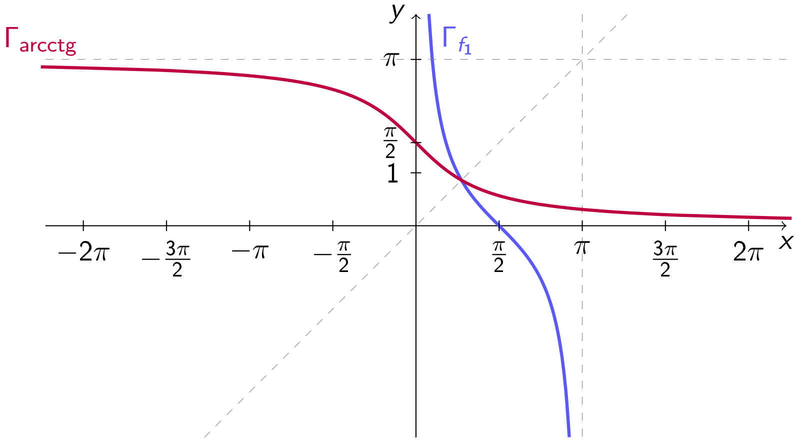

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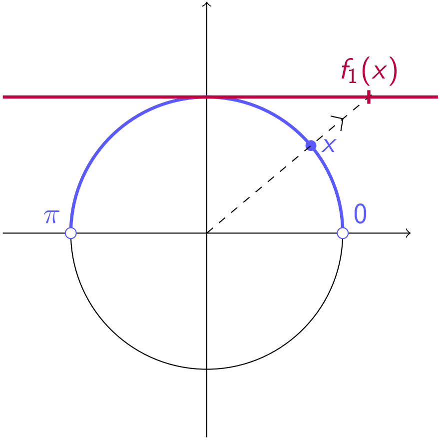
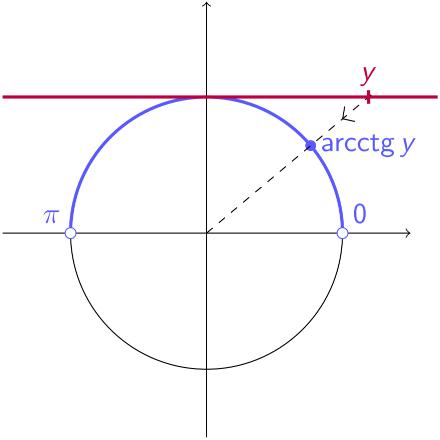
Zapamtimo:

$$\text{arcctg}\, y = \text{jedinstven } x \in \langle 0, \pi \rangle \text{ takav da je } \text{ctg}\, x = y.$$

Funkcija arcctg na brojevnoj kružnici





$f_1 : \langle 0, \pi \rangle \to \mathbb{R}$

$\text{arcctg} : \mathbb{R} \to \langle 0, \pi \rangle$

Zadatak 16

Odredite prirodnu domenu funkcije

$$f(x) := \operatorname{ctg} x \cdot \arcsin\left(1 - x^2\right). \tag{1}$$

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- $x \in \mathcal{D}_{\text{ctg}} = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$

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$$\overset{+1}{\Leftrightarrow} \quad 0 \leq x^2 \leq 2$$

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$$\Leftrightarrow \quad x^2 \leq 2$$

$$\Leftrightarrow \quad x^2 - 2 \leq 0$$

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$$\begin{aligned}
&\overset{+1}{\Leftrightarrow} \quad 0 \leq x^2 \leq 2 \\
&\Leftrightarrow \quad x^2 \leq 2 \\
&\Leftrightarrow \quad x^2 - 2 \leq 0 \\
&\Leftrightarrow \quad x \in \left[-\sqrt{2}, \sqrt{2}\right].
\end{aligned}$$

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$$\begin{aligned} &\Leftrightarrow \quad -1 \leq x^2 - 1 \leq 1 \\ &\overset{+1}{\Leftrightarrow} \quad 0 \leq x^2 \leq 2 \\ &\Leftrightarrow \quad x^2 \leq 2 \\ &\Leftrightarrow \quad x^2 - 2 \leq 0 \\ &\Leftrightarrow \quad x \in \left[-\sqrt{2}, \sqrt{2}\right]. \end{aligned}$$

Ovi su uvjeti ekvivalentni uvjetu

$$x \in \left[-\sqrt{2}, \sqrt{2}\right] \setminus \{k\pi : k \in \mathbb{Z}\} = [-\sqrt{2}, \sqrt{2}] \setminus \{0\}.$$

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Odredite prirodnu domenu funkcije

$$f(x) := \operatorname{ctg} x \cdot \arcsin\left(1 - x^2\right). \tag{1}$$

Rješenje. Desna strana formule (1) definirana je za neki $x \in \mathbb{R}$ ako i samo ako x zadovoljava sljedeće uvjete:

- $x \in \mathcal{D}_{\text{ctg}} = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$
- $x^2 - 1 \in \mathcal{D}_{\text{arcsin}} = [-1, 1]$

$$\begin{aligned} &\Leftrightarrow \quad -1 \leq x^2 - 1 \leq 1 \\ &\overset{+1}{\Leftrightarrow} \quad 0 \leq x^2 \leq 2 \\ &\Leftrightarrow \quad x^2 \leq 2 \\ &\Leftrightarrow \quad x^2 - 2 \leq 0 \\ &\Leftrightarrow \quad x \in \left[-\sqrt{2}, \sqrt{2}\right]. \end{aligned}$$

Ovi su uvjeti ekvivalentni uvjetu

$$x \in \left[-\sqrt{2}, \sqrt{2}\right] \setminus \{k\pi : k \in \mathbb{Z}\} = [-\sqrt{2}, \sqrt{2}] \setminus \{0\}.$$

Dakle, $\mathcal{D}_f = [-\sqrt{2}, \sqrt{2}] \setminus \{0\}$.

Zadatak 17(a)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$

Zadatak 17(a)

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Rješenje. Imamo

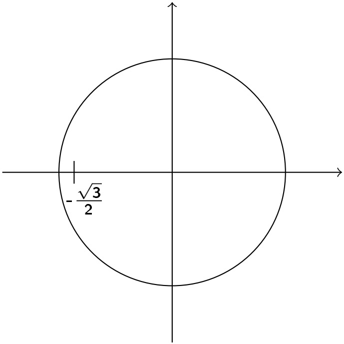
$$\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow$$

Zadatak 17(a)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\cos\left(2x-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}.$$



Rješenje. Imamo

$$\cos\left(2x-\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow$$

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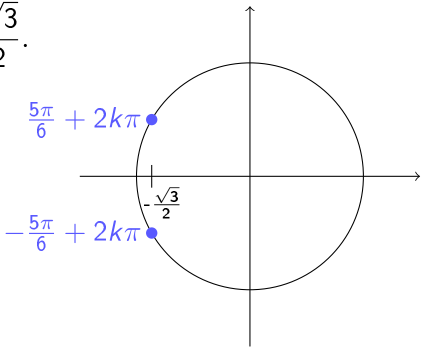

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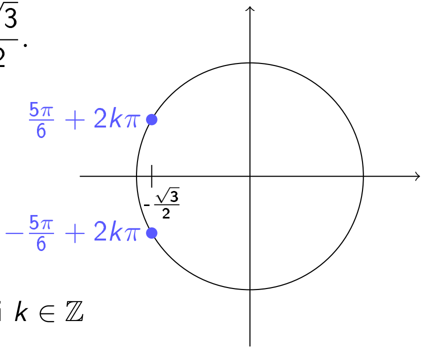


Rješenje. Imamo

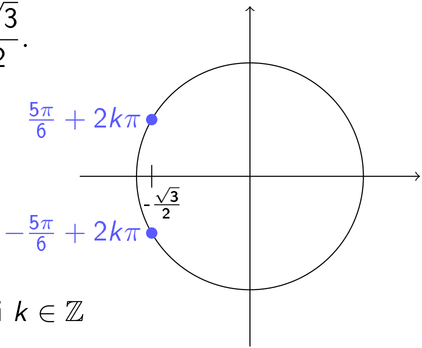
$$\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \quad 2x - \frac{\pi}{3} = \pm\frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z}$$

Zadatak 17(a)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$



Rješenje. Imamo

$$\begin{aligned}
&\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\
\Leftrightarrow \quad & 2x - \frac{\pi}{3} = \pm\frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow \quad & 2x = \frac{\pi}{3} \pm \frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z}
\end{aligned}$$

Zadatak 17(a)

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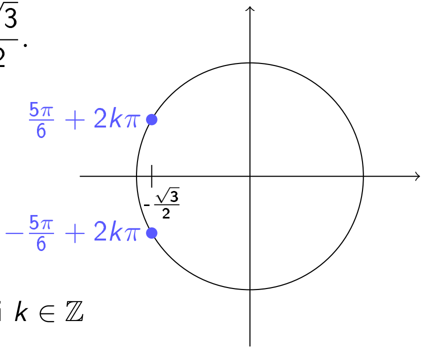


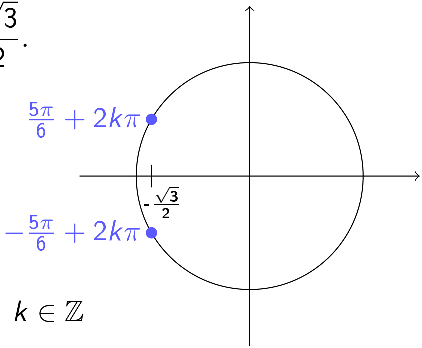
Rješenje. Imamo

$$\begin{aligned}
&\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\
\Leftrightarrow\quad & 2x - \frac{\pi}{3} = \pm\frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow\quad & 2x = \frac{\pi}{3} \pm \frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow\quad & x = \frac{\pi}{6} \pm \frac{5\pi}{12} + k\pi \text{ za neki } k \in \mathbb{Z}
\end{aligned}$$

Zadatak 17(a)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$



Rješenje. Imamo

$$\begin{aligned}
&\cos\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\
\Leftrightarrow \quad & 2x - \frac{\pi}{3} = \pm\frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow \quad & 2x = \frac{\pi}{3} \pm \frac{5\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow \quad & x = \frac{\pi}{6} \pm \frac{5\pi}{12} + k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow \quad & x \in \left\{\frac{7\pi}{12} + k\pi : k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{4} + k\pi : k \in \mathbb{Z}\right\}.
\end{aligned}$$

Zadatak 17(b)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1.$$

Zadatak 17(b)

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$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1.$$

Rješenje. Imamo

$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1$$

$$\Leftrightarrow$$

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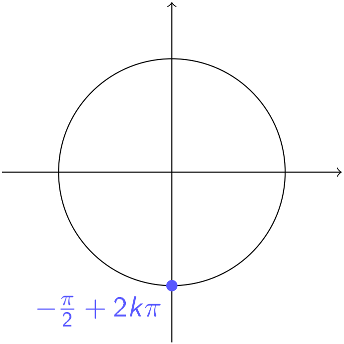
$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1.$$



Rješenje. Imamo

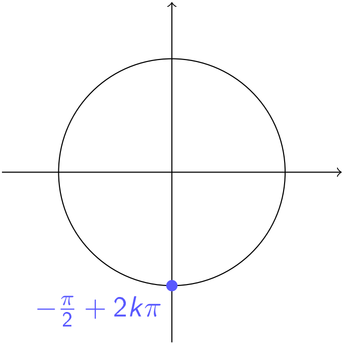
$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1$$

$$\Leftrightarrow$$

Zadatak 17(b)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right) = -1.$$



Rješenje. Imamo

$$\begin{aligned} &\sin\left(\frac{x}{2}+\frac{\pi}{6}\right) = -1 \\ \Leftrightarrow \quad &\frac{x}{2}+\frac{\pi}{6} = -\frac{\pi}{2}+2k\pi \text{ za neki } k\in\mathbb{Z} \end{aligned}$$

Zadatak 17(b)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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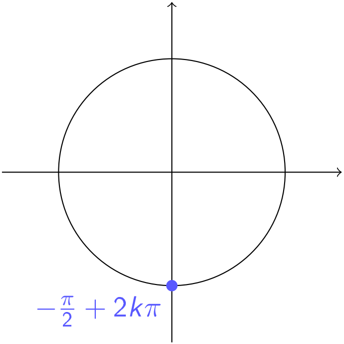

Rješenje. Imamo

$$\begin{aligned}
&\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1\\
\Leftrightarrow\quad & \frac{x}{2}+\frac{\pi}{6}=-\frac{\pi}{2}+2k\pi \text{ za neki } k\in\mathbb{Z}\\
\Leftrightarrow\quad & \frac{x}{2}=-\frac{2\pi}{3}+2k\pi \text{ za neki } k\in\mathbb{Z}
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Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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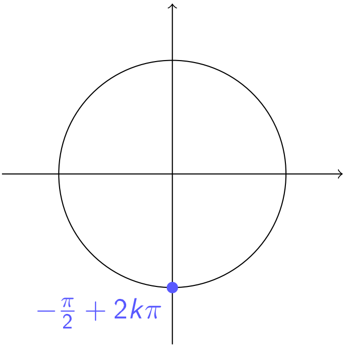

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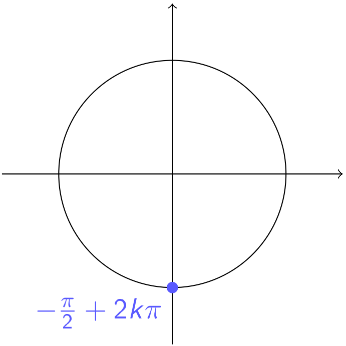
\Leftrightarrow\quad &\frac{x}{2}=-\frac{2\pi}{3}+2k\pi \text{ za neki } k\in\mathbb{Z}\\
\Leftrightarrow\quad &x=-\frac{4\pi}{3}+4k\pi \text{ za neki } k\in\mathbb{Z}
\end{aligned}$$

Zadatak 17(b)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)=-1.$$



Rješenje. Imamo

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\Leftrightarrow\quad &\frac{x}{2}+\frac{\pi}{6}=-\frac{\pi}{2}+2k\pi \text{ za neki } k\in\mathbb{Z}\\
\Leftrightarrow\quad &\frac{x}{2}=-\frac{2\pi}{3}+2k\pi \text{ za neki } k\in\mathbb{Z}\\
\Leftrightarrow\quad &x=-\frac{4\pi}{3}+4k\pi \text{ za neki } k\in\mathbb{Z}\\
\Leftrightarrow\quad &x\in\left\{-\frac{4\pi}{3}+4k\pi : k\in\mathbb{Z}\right\}.
\end{aligned}$$

Zadatak 17(c)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin 4x + \sin x = 0.$$

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Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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Rješenje. Koristeći da je za sve $x, y \in \mathbb{R}$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cdot \cos \frac{x-y}{2},$$

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dobivamo

$$\sin(4x) + \sin x = 0 \quad \Leftrightarrow \quad 2 \sin \frac{5}{2}x \cdot \cos \frac{3}{2}x = 0$$

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$$\Leftrightarrow \quad \sin \frac{5}{2}x = 0 \quad \text{ili} \quad \cos \frac{3}{2}x = 0$$

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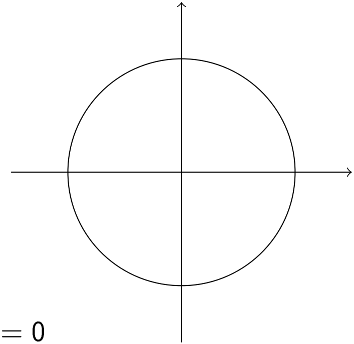

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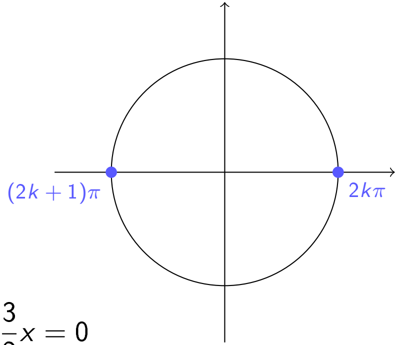

dobivamo

$$\sin(4x) + \sin x = 0 \quad \Leftrightarrow \quad 2 \sin \frac{5}{2}x \cdot \cos \frac{3}{2}x = 0$$

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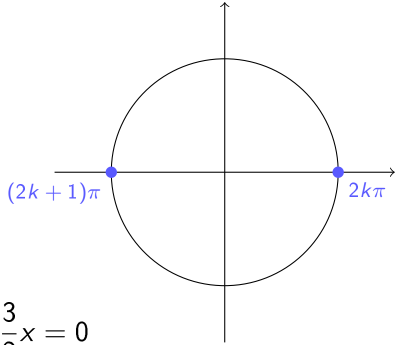

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&\Leftrightarrow \quad \frac{5}{2}x = k\pi \quad \text{ili}
\end{aligned}$$

Zadatak 17(c)

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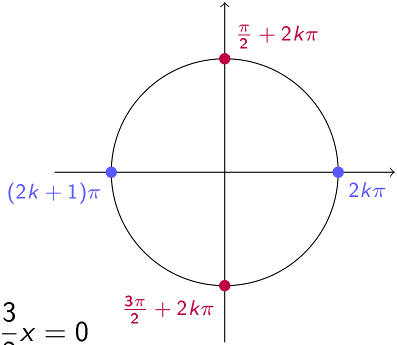
dobivamo

$$\begin{aligned}\sin(4x) + \sin x = 0 \quad &\Leftrightarrow \quad 2\sin\frac{5}{2}x\cdot\cos\frac{3}{2}x = 0\\ &\Leftrightarrow \quad \sin\frac{5}{2}x = 0 \quad \text{ili} \quad \cos\frac{3}{2}x = 0\\ &\Leftrightarrow \quad \frac{5}{2}x = k\pi \quad \text{ili}\end{aligned}$$



Zadatak 17(c)

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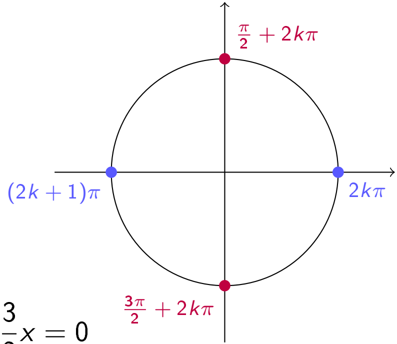

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&\Leftrightarrow \quad \sin\frac{5}{2}x = 0 \quad \text{ili} \quad \cos\frac{3}{2}x = 0 \\
&\Leftrightarrow \quad \frac{5}{2}x = k\pi \quad \text{ili} \quad \frac{3}{2}x = \frac{\pi}{2} + k\pi \text{ za neki } k \in \mathbb{Z}
\end{aligned}$$

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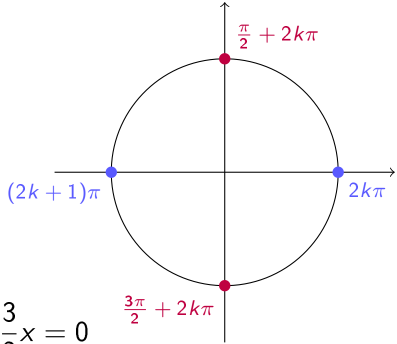

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&\Leftrightarrow \quad \sin\frac{5}{2}x = 0 \quad \text{ili} \quad \cos\frac{3}{2}x = 0 \\
&\Leftrightarrow \quad \frac{5}{2}x = k\pi \quad \text{ili} \quad \frac{3}{2}x = \frac{\pi}{2} + k\pi \text{ za neki } k \in \mathbb{Z} \\
&\Leftrightarrow \quad x = \frac{2k\pi}{5} \quad \text{ili} \quad x = \frac{\pi}{3} + \frac{2k\pi}{3} \text{ za neki } k \in \mathbb{Z}
\end{aligned}$$

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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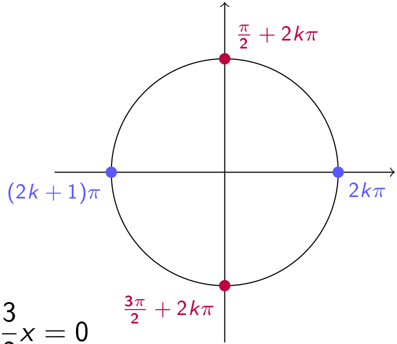

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&\Leftrightarrow \quad \frac{5}{2}x = k\pi \quad \text{ili} \quad \frac{3}{2}x = \frac{\pi}{2} + k\pi \text{ za neki } k \in \mathbb{Z}\\
&\Leftrightarrow \quad x = \frac{2k\pi}{5} \quad \text{ili} \quad x = \frac{\pi}{3} + \frac{2k\pi}{3} \text{ za neki } k \in \mathbb{Z}\\
&\Leftrightarrow \quad x \in \left\{\frac{2k\pi}{5} : k \in \mathbb{Z}\right\} \cup \left\{\frac{\pi}{3} + \frac{2k\pi}{3} : k \in \mathbb{Z}\right\}.
\end{aligned}$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin^2 x + 3\sin x \cdot \cos x + 2\cos^2 x = 0.$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin^2 x + 3\sin x \cdot \cos x + 2\cos^2 x = 0.$$

Rješenje. Dijeljenjem jednadžbe sa $\cos^2 x$ (nakon provjere da $\cos x = 0$ ne daje rješenje) dobivamo ekvivalentnu jednadžbu

$$\operatorname{tg}^2 x + 3\operatorname{tg} x + 2 = 0$$

Zadatak 17(d)

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$$\Leftrightarrow \quad (\text{supst. } t = \operatorname{tg} x) \quad \Leftrightarrow \quad t^2 + 3t + 2 = 0$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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$$\Leftrightarrow \quad (\text{supst. } t = \operatorname{tg} x) \quad \Leftrightarrow \quad t^2 + 3t + 2 = 0$$

$$\Leftrightarrow \quad (t+1)(t+2) = 0$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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$$\Leftrightarrow \quad t = -1 \quad \text{ili} \quad t = -2$$

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Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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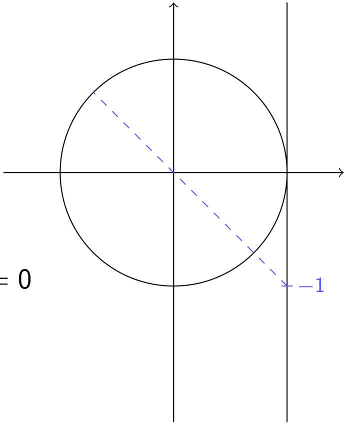
$$\Leftrightarrow \quad \operatorname{tg} x = -1 \quad \text{ili} \quad \operatorname{tg} x = -2$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$\sin^2 x + 3\sin x \cdot \cos x + 2\cos^2 x = 0.$$



Rješenje. Dijeljenjem jednadžbe sa $\cos^2 x$ (nakon provjere da $\cos x = 0$ ne daje rješenje) dobivamo ekvivalentnu jednadžbu

$$\operatorname{tg}^2 x + 3\operatorname{tg} x + 2 = 0$$

$$\Leftrightarrow \quad (\text{supst. } t = \operatorname{tg} x) \quad \Leftrightarrow \quad t^2 + 3t + 2 = 0$$

$$\Leftrightarrow \quad (t+1)(t+2) = 0$$

$$\Leftrightarrow \quad t = -1 \quad \text{ili} \quad t = -2$$

$$\Leftrightarrow \quad \operatorname{tg} x = -1 \quad \text{ili} \quad \operatorname{tg} x = -2$$

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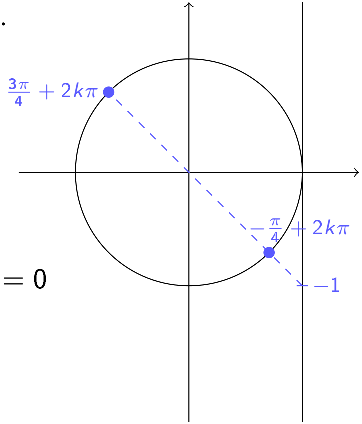

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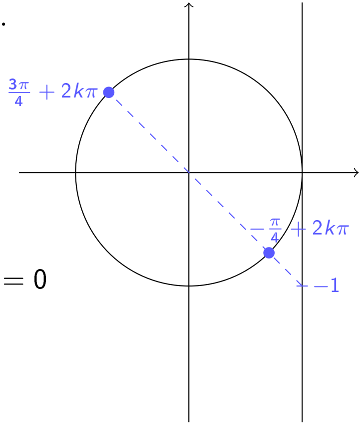

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$$\Leftrightarrow \quad x = -\frac{\pi}{4} + k\pi \text{ za neki } k \in \mathbb{Z} \quad \text{ili}$$

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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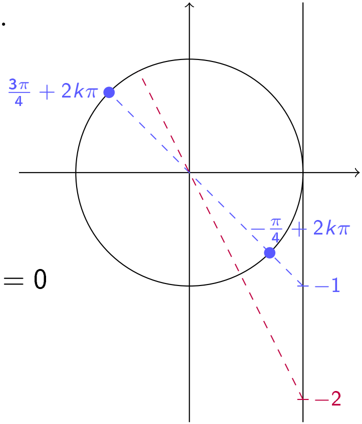

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$\Leftrightarrow \quad \operatorname{tg} x = -1 \quad$ ili $\quad \operatorname{tg} x = -2$

$\Leftrightarrow \quad x = -\dfrac{\pi}{4} + k\pi$ za neki $k \in \mathbb{Z} \quad$ ili

Zadatak 17(d)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

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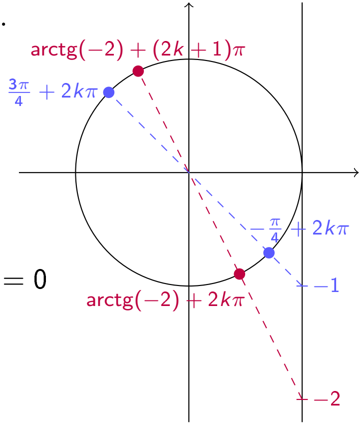

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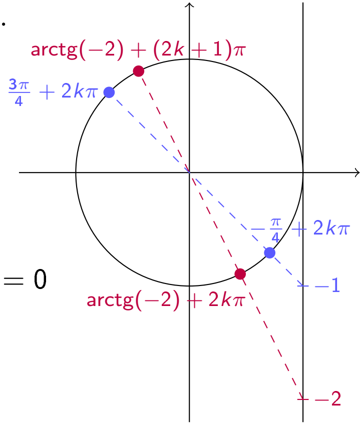

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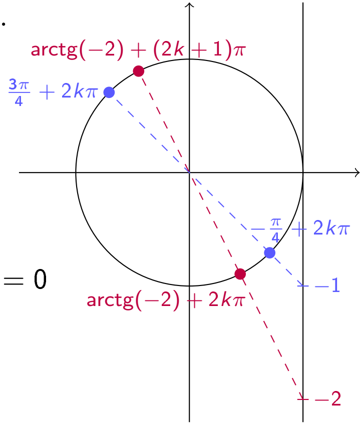

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$$\Leftrightarrow \quad x \in \left\{-\frac{\pi}{4} + k\pi : k \in \mathbb{Z}\right\} \cup \{\operatorname{arctg}(-2) + k\pi : k \in \mathbb{Z}\}.$$

Zadatak 17(e)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$2\sin x - 3\cos x = 1.$$

Zadatak 17(e)

Odredite sva rješenja (u $\mathbb{R}$) jednadžbe

$$2\sin x - 3\cos x = 1.$$

Rješenje. Jednadžbe

$$a\sin x + b\cos x = c,$$

gdje su $a, b, c \in \mathbb{R} \setminus \{0\}$, najlakše je riješiti uvođenjem tzv. **univerzalne supstitucije**

$$t = \operatorname{tg}\frac{x}{2} \quad \rightsquigarrow \quad \sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}.$$

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$$2 \cdot \frac{2t}{1+t^2} - 3 \cdot \frac{1-t^2}{1+t^2} = 1 \quad \overset{\text{sami}}{\Leftrightarrow} \quad t = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$$

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! Na kraju trebamo provjeriti jesu li i brojevi $x = (2k+1)\pi$, $k\in\mathbb{Z}$ (za koje $\operatorname{tg}\frac{x}{2}$ nije definiran) rješenja.

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Zadatak 18(a)

Riješite nejednadžbu

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}.$$

Zadatak 18(a)

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Rješenje. Imamo

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow$$

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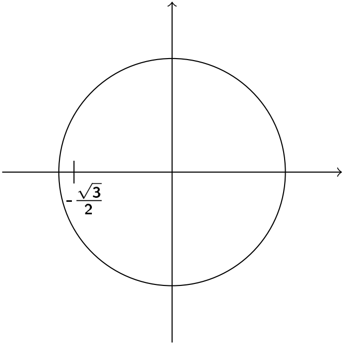

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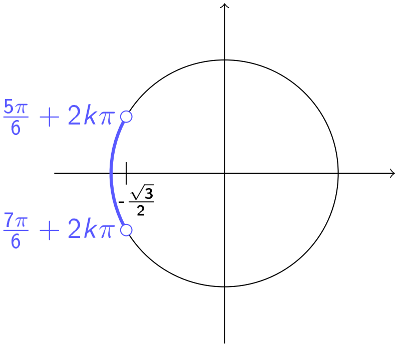

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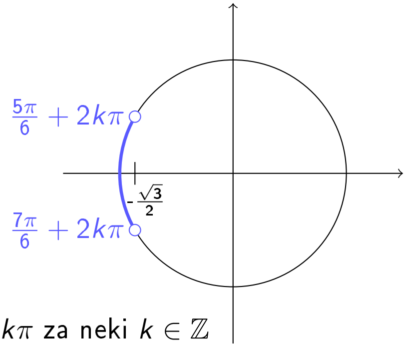

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}$$

$$\overset{\text{slika}}{\Leftrightarrow} \quad \frac{5\pi}{6} + 2k\pi < 2x - \frac{\pi}{3} < \frac{7\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z}$$

Zadatak 18(a)

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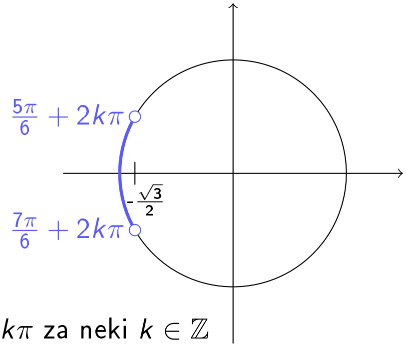

Rješenje. Imamo

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}$$

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$$\overset{+\frac{\pi}{3}}{\Leftrightarrow} \quad \frac{7\pi}{6} + 2k\pi < 2x < \frac{3\pi}{2} + 2k\pi \text{ za neki } k \in \mathbb{Z}$$

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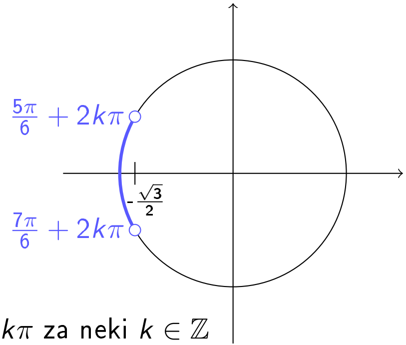
Riješite nejednadžbu

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}.$$



Rješenje. Imamo

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}$$

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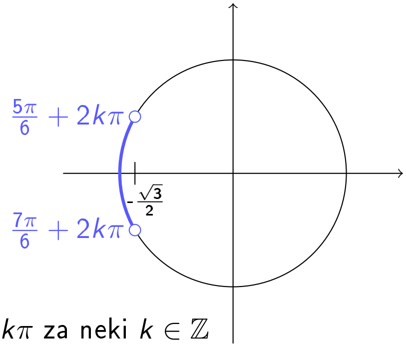
$$\overset{+\frac{\pi}{3}}{\Leftrightarrow} \quad \frac{7\pi}{6} + 2k\pi < 2x < \frac{3\pi}{2} + 2k\pi \text{ za neki } k \in \mathbb{Z}$$

$$\overset{\cdot\frac{1}{2}}{\Leftrightarrow} \quad \frac{7\pi}{12} + k\pi < x < \frac{3\pi}{4} + k\pi \text{ za neki } k \in \mathbb{Z}$$

Riješite nejednadžbu

$$\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2}.$$



Rješenje. Imamo

$$\begin{aligned}
&\cos\left(2x - \frac{\pi}{3}\right) < -\frac{\sqrt{3}}{2} \\
\overset{\text{slika}}{\Leftrightarrow}\quad & \frac{5\pi}{6} + 2k\pi < 2x - \frac{\pi}{3} < \frac{7\pi}{6} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\overset{+\frac{\pi}{3}}{\Leftrightarrow}\quad & \frac{7\pi}{6} + 2k\pi < 2x < \frac{3\pi}{2} + 2k\pi \text{ za neki } k \in \mathbb{Z} \\
\overset{\cdot\frac{1}{2}}{\Leftrightarrow}\quad & \frac{7\pi}{12} + k\pi < x < \frac{3\pi}{4} + k\pi \text{ za neki } k \in \mathbb{Z} \\
\Leftrightarrow\quad & x \in \bigcup_{k\in\mathbb{Z}} \left\langle \frac{7\pi}{12} + k\pi, \frac{3\pi}{4} + k\pi \right\rangle.
\end{aligned}$$

Zadatak 18(b)

Riješite nejednadžbu

$$\operatorname{ctg} x \geq 1.$$

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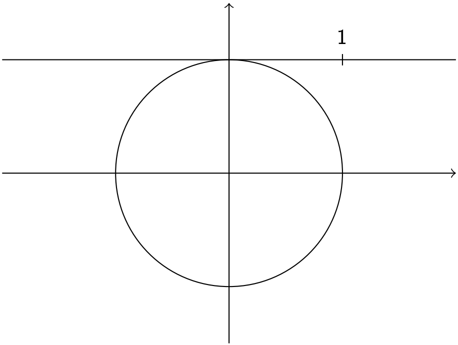
Rješenje.



Zadatak 18(b)

Riješite nejednadžbu

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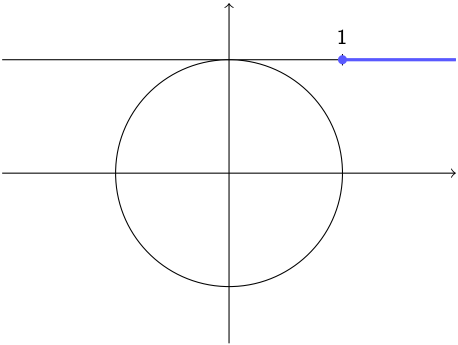
Rješenje.



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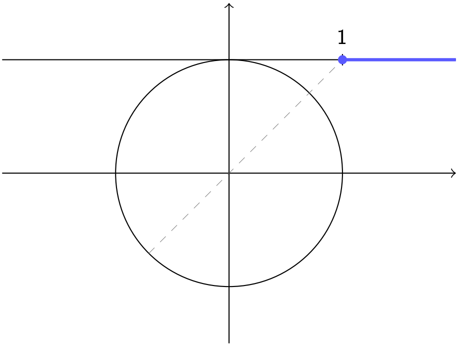
Riješite nejednadžbu

$$\operatorname{ctg} x \geq 1.$$

Rješenje.

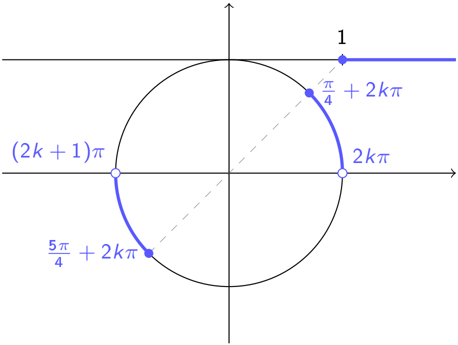


Zadatak 18(b)

Riješite nejednadžbu

$$\operatorname{ctg} x \geq 1.$$

Rješenje.

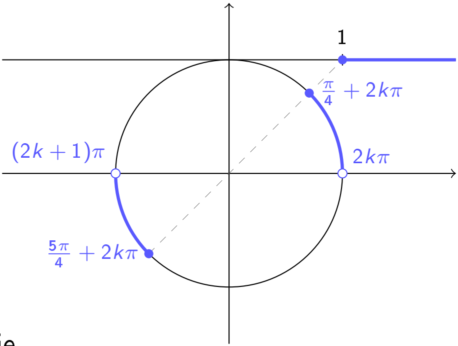


Zadatak 18(b)

Riješite nejednadžbu

$$\operatorname{ctg} x \geq 1.$$

Rješenje.



Sa slike vidimo da je

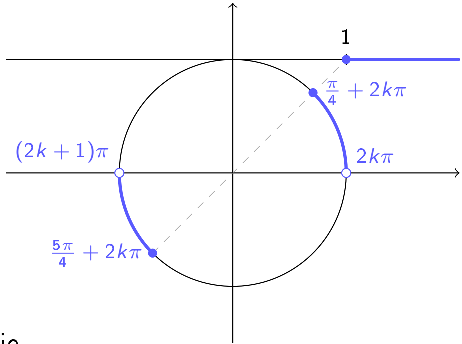
$$\operatorname{ctg} x \geq 1 \;\Leftrightarrow\; x \in \bigcup_{k\in\mathbb{Z}} \left(\left\langle 2k\pi, \frac{\pi}{4} + 2k\pi \right] \cup \left\langle (2k+1)\pi, \frac{\pi}{4} + (2k+1)\pi \right] \right)$$

Zadatak 18(b)

Riješite nejednadžbu

$$\operatorname{ctg} x \geq 1.$$

Rješenje.



Sa slike vidimo da je

$$\operatorname{ctg} x \geq 1 \iff x \in \bigcup_{k \in \mathbb{Z}} \left(\left\langle 2k\pi, \frac{\pi}{4} + 2k\pi \right] \cup \left\langle (2k+1)\pi, \frac{\pi}{4} + (2k+1)\pi \right] \right)$$

$$\iff x \in \bigcup_{k \in \mathbb{Z}} \left\langle k\pi, \frac{\pi}{4} + k\pi \right].$$